Testing of an edge thermal instability stabilization model for the low-to-high mode power threshold

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ABSTRACT

A test of a new model for the low-to-high (L-H) mode power threshold, based on the stabilization of edge thermal instabilities, is made by comparison with a set of DIII-D [J. L. Luxon, Nucl. Fusion, 42, 614, 2002] discharges at times just prior to a L-H transition. Agreement is found between the measured power crossing the separatrix just prior to the L-H transition and the predicted power threshold for the stabilization of transport enhancing thermal instabilities.

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I. INTRODUCTION

It was shown recently[1] that there is a critical, threshold non-radiative heat flux through the plasma edge above which thermal instabilities with short radial wavelengths[2] are stabilized. The predicted phenomena--a decrease in the values of both edge temperature gradient scale length and the heat conductivity associated with the thermal instabilities as the power flux approached the threshold value from below and the sharp decrease in both quantities as the threshold was crossed--were suggestive of the low-to-high (L-H) transition in tokamaks. The predicted phenomena, as the power flux approached the threshold value from above, were similarly suggestive of the H-L back transition.

A first test of this prediction of a threshold heat flux for the H-L back transition against data from DIII-D[3] was recently published[4] for a set of 'density limit' shots in which the density was increased in H-mode discharges by gas fueling until a H-L mode back transition took place. The increasing density produced increasing core radiation and, at constant heating power, decreasing non-radiative power flowing outward across the separatrix. Good agreement was obtained between the predicted and experimental values of the non-radiative power crossing the separatrix at the time of the H-L back transition for a set of shots with high radiative power fraction.

The purpose of this paper is to report the results of a similar comparison of predicted and measured values of the non-radiative power crossing the separatrix at the time of the L-H transition for a representative set of DIII-D shots in which the radiative power fraction is small at the time of the transition.

II. L-H POWER THRESHOLD MODEL

The L-H mode power threshold model[1] that we wish to test has the following elements: 1) a model[2] for the growth rate of thermal instabilities with short radial wavelengths in the edge pedestal region, or 'transport barrier'; 2) an enhancement of transport in the pedestal when thermal instabilities are growing; and 3) the conventional transport heat conduction closure relation among heat fluxes, temperature gradients and transport coefficients.

A linear analysis of the stability of the plasma particle, momentum and energy balance equations in the edge pedestal against two-dimensional (r-⊥) coupled density, velocity and temperature perturbations with radial wavelength k_r^{-1} leads to a dispersion relation from which the growth rates (real parts of ω) of such modes can be calculated[2]. Since particles move rapidly along field lines the thermal instability is not localized poloidally, although it is localized radially. The flux surface average of the poloidally dependent edge neutral concentration is used to account for the fact that particles move rapidly along field lines during the time required for the growth of a thermal instability. The calculation has been carried out in the weak- and strong-equilibration limits. In the weak ion-electron equilibration limit, the growth rates of local thermal instabilities associated with the ion and electron energy balances are decoupled and both may be written in the general form

$$\omega = -\frac{2}{3}\left(\chi\left(\nu L_T^{-2} + k_r^2\right) + \frac{5}{2}\nu\frac{\Gamma_\perp}{n}L_T^{-1} - \alpha\right) \tag{1}$$

where the first two terms represent the generally stabilizing effect of heat conduction and convection, respectively, with $L_T^{-1} = (-dT/dr)/T$ for the species in question, $\Gamma_\perp$ being the outward ion or electron particle flux, and ν characterizing the temperature dependence of the underlying thermal conductivity for that species, $\chi^0 \sim T^\nu$. The α-terms represent the generally destabilizing atomic physics and impurity cooling terms in the respective growth rates for the ions

$$\alpha_i = \frac{5}{2}(\nu-1)\nu_{ion} + \frac{3}{2}\nu_{at}^c\left(\nu - \left[1 + \frac{T_i}{\nu_{at}^c}\frac{\partial \nu_{at}^c}{\partial T_i}\right]\right) - \frac{1}{n}\left(\nu\frac{H_i}{T_i} - \frac{\partial H_i}{\partial T}\right) \tag{2a}$$

and for the electrons

$$\begin{aligned}\alpha_e = n_z\left(\frac{\nu L_z}{T_e} - \frac{\partial L_z}{\partial T_e}\right) + \nu_{ion}\left\{\frac{5}{2}(\nu-1) + \nu\frac{E_{ion}}{T_e} - \left(\frac{3}{2} + \frac{E_{ion}}{T_e}\right)\frac{T_e}{\nu_{ion}}\frac{\partial \nu_{ion}}{\partial T_e}\right\} \\ - \frac{1}{n}\left(\nu\frac{H_e}{T_e} - \frac{\partial H_e}{\partial T_e}\right)\end{aligned} \tag{2b}$$

The terms ν_{ion} and ν_{at} are the neutral ionization frequency in the pedestal region and the frequency of charge-exchange and elastic scattering events involving 'cold' neutrals that have not previously undergone such an event in the pedestal region. E_{ion} is the ionization energy, and n_z and L_z are the density and radiative emissivity of impurities in the edge pedestal region. H represents any additional heating or cooling in the pedestal.

Since we are considering edge modes driven by atomic physics and impurity radiative cooling, these modes should have a radial extent inward from the separatrix on the order of the mean free path for neutral penetration or for the impurity electrons to be stripped beyond the most highly radiative states, Δ_{edge}, which is a few cm in DIII-D. Thus, the radial wavelengths of the instabilities should be the harmonics of Δ_{edge}; i.e. $k_r^{-1} = \Delta_{edge}/m$, m = 1, 2, 3…. The resulting expression for the power threshold will turn out to be relatively insensitive to the exact value of k_r^{-1} in the 1-10 cm range of interest.

A number of different types of turbulence lead[5] to the simple and frequently used estimate (Kadomtsev's connection length estimate) that the incremental transport associated with instabilities with linear growth rate ω and wave number k_r is $\Delta\chi = \omega k_r^{-2}$. We write the thermal diffusivity as an underlying term χ^0 that is present in the absence of thermal instabilities plus a contribution from thermal instabilities

$$\chi = \chi^0 + C_\chi \omega\, k_r^{-2} H(\omega > 0) \tag{3}$$

where C_χ is an order unity constant and the Heaviside function H = 1 when $\omega > 0$ and H = 0 when $\omega \le 0$.

From Eq. (1), there is clearly a threshold value of L_T^{-1} above which $\omega < 0$. Relating this threshold value of L_T^{-1} to the total heat flux $Q_\perp$ by using the general form for the conductive heat flux transport closure relation

$$\left(\frac{Q_\perp}{nT} - \frac{5}{2}\frac{\Gamma_\perp}{n}\right) = \chi L_T^{-1} \tag{4}$$

results in an expression for a threshold value of the non-radiative power flux through the edge pedestal above which thermal instabilities do not exist (i.e. $\omega < 0$)

$$\left(\frac{Q_\perp}{nT}\right)_{thresh} = \left(\frac{5}{4}\frac{\Gamma_\perp}{n}\right)\left[\sqrt{1 + \frac{\left(\chi^0\left(\alpha - \chi^0 k_r^2\right)/\nu\right)}{\left(\frac{5}{4}\frac{\Gamma_\perp}{n}\right)^2}} + 1\right] \tag{5}$$

It has been shown[1] that for a non-radiative power flux below the threshold given by Eq. (5), the quadratic (in L_T or in χ/χ^0) equation obtained by combining Eqs. (1) and (3) has two roots and that the larger root decreases as $(Q_\perp/nT)$ approaches the threshold value from below. As $(Q_\perp/nT)$ increases above the threshold, the larger root of χ decreases sharply to coalesce with the smaller root with value χ^0, and L_T decreases sharply (the temperature gradient becomes sharply steeper) to the value given by Eq. (4). Conversely, as $(Q_\perp/nT)$ approaches the threshold from above, Eq. (4) shows that the temperature gradient will become progressively less steep, and then when $(Q_\perp/nT)$ drops below the threshold value the thermal conductivity will increase sharply and the temperature gradient will decrease sharply, to the larger root of the quadratic equation that obtains below the threshold. These predicted phenomena are suggestive of the L-H and H-L transitions.

For our purpose in this paper—to test the power flux threshold formula of Eq. (5) against the measured non-radiative power crossing the separatrix just prior to the time that an L-mode shot made a transition into H-mode—it is convenient to convert Eq. (5) to a power threshold for the respective ion or electron thermal instability

$$P_{thresh} = \frac{5}{4}\Gamma_\perp T\, A_{sep}\left[\sqrt{1 + \frac{\left(\chi^0\left(\alpha - \chi^0 k_r^2\right)/\nu\right)}{\left(\frac{5}{4}\frac{\Gamma_\perp}{n}\right)^2}} + 1\right] \tag{6}$$

where A_{sep} is the area of the plasma surface at the separatrix.

In the strong equilibration case, Eq.(1) also obtains but now with χ and α being the average of the ion and electron values, and the subsequent development leads again to Eq. (6) as the common power threshold for the ion and electron thermal instabilities.

III. COMPARISON WITH DIII-D L-H TRANSITION DATA

We selected a representative set of shots with a range of operating conditions for examination and evaluated Eq. (6) for the plasma conditions measured at a time just prior to the L-H transition for comparison with the measured power crossing the separatrix at the same time. As a control, we performed the same calculations at a later time in the middle of a long H-mode phase for one of the shots (97979 @ 3250 ms). Some relevant parameters for these shots, as well as values ofthe experimental power crossing the separatrix and the threshold power prediction of Eq. (6) are given in Table 1.

A power balance on the plasma was performed by adding the known neutral beam power to the measured ohmic power and subtracting the measured radiation power from within the separatrix and the measured rate of increase in the thermal energy content of the plasma. The resulting power crossing the separatrix immediately before the L-H transition is shown as P_{sep}^{ex} in Table 1—the spread in values arises from the estimated uncertainty in the measurement of the power radiated from inside the separatrix. The measured power radiated from within the separatrix was about 10% or less of the total heating power for these shots just prior to the L-H transition.

In order to evaluate Eq. (6) for comparison with P_{sep}^{ex}, we must calculate the fueling of the edge pedestal and the core plasma by recycling neutrals in order to evaluate the outward particle flux $\Gamma_{\perp}$ from particle balance on the core and in order to evaluate the ionization, charge-exchange and elastic scattering frequencies that appear in the α-terms of Eqs. (2). This calculation requires the modeling of the plasma and a neutral recycling calculation.

The core and divertor/scrape-off layer plasma were modeled as described in Ref. 6 for the purpose of providing a background plasma for the neutral transport calculation and for calculating particle and heat fluxes into the pedestal region from the core. The measured edge plasma densities were used in the neutral attenuation calculations, and the neutral source from the wall was adjusted so that the calculated line average density (from particle balance using a particle confinement time measured in pellet 'die-away' experiments) matched the measured value, in order to calibrate the calculation of the neutral influx contribution to $\Gamma_{\perp}$ to experiment. This neutral transport calculation methodology has been corroborated previously by comparison with a measurement of neutral densities in DIII-D[7]. The measured energy confinement time was used in the global plasma energy balance to calculate the plasma core average temperature, and the calculated core radiation was adjusted to match the measured radiated power, in order to calibrate the total heat flux ($Q\perp$) through the pedestal to experiment. The calculated radiation in the divertor and scrape-off layer were also adjusted to match experiment in order to calibrate the background divertor plasma used in the neutral recycling calculation.

It is also necessary to specify the heat conductivities and their temperature dependence, ν, and to specify k_r in order to evaluate the terms under the radical in Eqs. (5) or (6) . The α-terms have magnitude of order 10^3 /s for these shots, and the dominant contribution to these terms is from the neutral ionization and charge-exchange terms. Since probably $0.1 \leq \chi^0 \leq 1$ m^2/s, the $\chi^0 k_r^2$ term in Eq. (6) is not important for $1 < k_r^{-1} < 10$ cm, and we do not need to be more specific about k_r. (The implication is that all modes with k_r in this range have approximately the same threshold power.) Consideration of various anomalous transport theories indicates[8] that $3/2 \leq \nu \leq 7/2$. Although the second term under the radical in Eq. (6) is comparable to and sometimes larger than unity for these shots, we find that the threshold power is relatively insensitive (order a few %) to variation of χ^0 and ν over these ranges.

In addition to the α-terms, the most important quantity in determining the power threshold is the outward ion flux across the pedestal. This is determined by subtracting the measured rate at which the total number of electrons in the plasma is increasing from the known rate at which ions are being deposited by neutral beam injection plus the calculated rate (see above) at which neutrals are flowing inward across the separatrix. Particle fluxes of a few times 10^{20} /m^2s were calculated for these shots.

Since we do not know the split of P_{sep}^{ex} between the ion and electron channels, we can not compare $P_{sep}^{i,e}$ with $P_{thr}^{i,e}$. However, if $P_{sep}^{ex} < P_{thr}^{e}+P_{thr}^{i}$ the power crossing the separatrix in either the ion or the electron channel, or both, is less than the threshold power for thermal instability onset in the respective channel or channels. Thus, we take $P_{sep}^{ex} \approx P_{thr}^{e}+P_{thr}^{i}$ as an approximate prediction of a L-H transition. The sum of the weak-equilibration values of the ion and electron thermal instability thresholds are shown in Table 1; the strong equilibration values are similar. $P_{sep}^{ex} \approx P_{thr}^{e}+P_{thr}^{i}$ at times just prior to when a L-H transition was observed in the first four shots in Table 1. For the fifth 'control' shot (97979 @ 3250 ms), which was in the middle of a long H-mode phase at 3250 ms, $P_{sep}^{ex} >> P_{thr}^{e}+P_{thr}^{i}$, indicating that this condition was not predicted to be near either an L-H or H-L transition . The conduction part of the total heat flux in these shots was calculated from power balance and subtraction of the convective flux to be 65-80% of the total heat flux across the separatrix.

As shown in Ref. (1), the threshold power expression of Eq. (5) or (6) resulted from imposing the condition that the growth rate for thermal instabilities was zero. This growth rate was determined from a competition between the stabilizing effects of heat conduction and convection (Γ) and the destabilizing effects of radiation and atomic physics cooling (α_i and α_e). The threshold power expression resulted from the requirement on the conductive heat flux for stability, for given levels of heat convection and radiation and atomic physics cooling, hence the dependence on the α_i, α_e and Γ. The values of the cooling terms, α, and the particle flux terms, Γ, that determine the threshold powers given in Table 1 varied over about a factor of 2, as may be seen by comparing the quantities (α_i, α_e, Γ, $\underline{P_{thr}}$) for three of the shots in Table 1: 92079 (2.2e3, 2.0e3, 4.1e20, <u>4.0</u>), 97979 (1.3e3, 1.1e3, 3.4e20, <u>2.2</u>), and 102456 (9.4e2, 8.2e2, 2.2e20, <u>1.5</u>), and the resulting threshold powers varied by almost a factor of 3. Clearly, both the α's and Γ are important in the determination of the threshold power. If we include the H-L back transition shots of Ref. (4), the threshold power expression of Eq. (6) now has been tested over a range of values of Γ that vary by about a factor of 4 and a range of α's that vary by about a factor of 5, and the resulting range of threshold powers vary by about a factor of 4.

IV. DISCUSSION

Good agreement has been found between the measured non-radiative power crossing the separatrix just prior to a L-H transition and the sum of the predicted threshold powers for thermal instability stabilization in the ion and electron power balances, for a set of shots with core radiative power fractions of 10% or less. We recall the previous finding[4] that the same power threshold expression predicts values in good agreement with measured non-radiative power crossing the separatrix just prior to a H-L back transition for a set of 'density limit' shots with core radiative power fractions of 20-40%. These findings combine to provide a strong suggestion that stabilization of thermal instabilities in the edge pedestal plays a major role in triggering the L-H transition and that destabilization plays a similar role in triggering the back H-L transition.

Even broader experimental support for the stabilization of thermal instabilities as a trigger mechanism for the L-H transition may be inherent in the recent finding[9] that edge gradients in temperature and pressure may be better control parameters for predicting the L-H transition than the edge values of the temperature or pressure. The temperature and pressure gradients, but not the electron density gradient, all measured in the region in which the H-mode pedestal ultimately formed, were found to increase during the L-mode phase in shots which made a H-mode transition. This is consistent with the predicted thermal instability stabilization due to increasing temperature gradients as the threshold power is approached that was discussed earlier in this paper and more fully in Ref. 1.

To put these results in perspective, we note that the L-H transition has been studied experimentally for more than a decade (e.g. Refs. 10-14) and that the reigning paradigm for the L-H transition that has emerged is the suppression of turbulent transport by the sheared ExB flow produced by a sharp gradient in the negative radial electric field just inside the separatrix. Triggering mechanisms previously put forward to account for the creation of this local radial electric field shear include orbit loss[15] and Stringer spin-up[16]. It has been suggested[1] that the reduced transport that occurs when the power threshold of Eq. (5) or (6) is exceeded produces a reduced particle flux across the separatrix (supported by D_α measurements) that in turn produces a positive poloidal rotation (as observed) that results via momentum balance in a negative radial electric field. Thus, the thermal instability suppression mechanism[1], the threshold power prediction of which was confirmed in this paper and in Ref. 4, provides another possible explanation for the trigger mechanism for L-H and H-L transitions.

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Table 1 Some DIII-D shots just prior to the L-H transition (R=1.71-1.79m, a=0.6m, κ=1.73-1.89, LSN divertor)

Shot #	*Time (ms)*	*I (MA)*	*B (T)*	P_{NB} *(MW)*	δ	n_{eped} *(e19/m*^{3}*)*	T_{eped} *(eV)*	P_{sep}^{exp} *(MW)*	P_{thr} *(MW)*
102456	1725	1.4	2.0	2.6	0.73	3.22	95	1.55-1.86	1.54
97979	1900	1.4	2.0	2.0	0.79	2.59	125	1.72-2.04	2.18
92079	2275	1.0	2.1	6.8	0.37	1.28	220	3.99-4.06	4.00
84027	2575	1.3	2.1	1.1	0.32	2.94	144	1.28-1.36	1.13
97979[a]	3250	1.4	2.0	6.5	0.79	6.35	525	4.64-4.96	2.59

[a] well into H-mode phase, not at the L-H transition—control case